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**MATHEMATICS  
HIGHER LEVEL  
PAPER 3 – STATISTICS AND PROBABILITY**

Monday 9 May 2011 (morning)

1 hour

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

The weights of the oranges produced by a farm may be assumed to be normally distributed with mean 205 grams and standard deviation 10 grams.

- (a) Find the probability that a randomly chosen orange weighs more than 200 grams. [2 marks]
- (b) Five of these oranges are selected at random to be put into a bag. Find the probability that the combined weight of the five oranges is less than 1 kilogram. [4 marks]
- (c) The farm also produces lemons whose weights may be assumed to be normally distributed with mean 75 grams and standard deviation 3 grams. Find the probability that the weight of a randomly chosen orange is more than three times the weight of a randomly chosen lemon. [5 marks]

2. [Maximum mark: 10]

The discrete random variable  $X$  has the distribution  $\text{Geo}(p)$ . The value of  $p$  is believed to be 0.3. In order to test this belief a random sample of 100 values of  $X$  was obtained, with the following results.

<b>Value of <math>X</math></b>	1	2	3	4	5	$\geq 6$
<b>Frequency</b>	35	18	16	11	10	10

- (a) State suitable hypotheses to test this belief. [1 mark]
- (b) Carry out a  $\chi^2$ -test at the 5 % significance level and, giving a reason, state your conclusion. [9 marks]

3. [Maximum mark: 17]

Ten friends try a diet which is claimed to reduce weight. They each weigh themselves before starting the diet, and after a month on the diet, with the following results.

Friend	A	B	C	D	E	F	G	H	I	J
Weight before (kg)	68.4	70.9	74.7	65.4	59.4	69.0	73.9	62.6	68.3	58.2
Weight after (kg)	66.2	67.4	70.4	65.9	55.2	69.2	71.4	59.9	68.2	58.9

- (a) Determine unbiased estimates of the mean and variance of the loss in weight achieved over the month by people using this diet. [5 marks]
- (b) (i) State suitable hypotheses for testing whether or not this diet causes a mean loss in weight.
- (ii) Determine the value of a suitable statistic for testing your hypotheses.
- (iii) Find the 1 % critical value for your statistic and state your conclusion. [6 marks]
- (c) One of the friends calculates a confidence interval for the mean loss in weight obtained by people using this diet for a month and he obtains [0.26, 3.36]. Find the confidence level of this interval. [6 marks]

4. [Maximum mark: 10]

The random variable  $X$  has a Poisson distribution with unknown mean  $\mu$ . It is required to test the hypotheses

$$H_0 : \mu = 3 \text{ against } H_1 : \mu \neq 3.$$

Let  $S$  denote the sum of 10 randomly chosen values of  $X$ . The critical region is defined as  $(S \leq 22) \cup (S \geq 38)$ .

- (a) Calculate the significance level of the test. [5 marks]
- (b) Given that the value of  $\mu$  is actually 2.5, determine the probability of a Type II error. [5 marks]

5. [Maximum mark: 12]

- (a) The random variable  $X$  has the negative binomial distribution  $NB(3, p)$ . Let  $f(x)$  denote the probability that  $X$  takes the value  $x$ .

- (i) Write down an expression for  $f(x)$ , and show that

$$\ln f(x) = 3 \ln \left( \frac{p}{1-p} \right) + \ln(x-1) + \ln(x-2) + x \ln(1-p) - \ln 2.$$

- (ii) State the domain of  $f$ .

- (iii) The domain of  $f$  is extended to  $]2, \infty[$ . Show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x-1} + \frac{1}{x-2} + \ln(1-p). \quad [7 \text{ marks}]$$

- (b) Jo has a biased coin which has a probability of 0.35 of showing heads when tossed. She tosses this coin successively and the 3<sup>rd</sup> head occurs on the  $Y^{\text{th}}$  toss. Use the result in part (a)(iii) to find the most likely value of  $Y$ . [5 marks]